Exam. Code : 211001
Subject Code : 3835

M.Sc. (Mathematics) 1st Semester

COMPLEX ANALYSIS

Paper—Math-552

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **five** questions in all, selecting at least **one** from each Section. All questions carry equal marks.

SECTION—A

1. Find a necessary and sufficient condition for a complex valued function to be analytic.

2. (a) Define complex line integral. Evaluate the integral

$$\int_{-2+i}^{5+3i} z^2 dz .$$

(b) State and prove Cauchy's integral formula for first order derivative.

SECTION—B

3. (a) State and prove Cauchy's inequality.

(b) If w=f(z) represents a conformed transformation of a domain D in the z-plane into a domain D' of the w-plane then show that f(z) is an analytic function of z in D'.

4. (a) Find the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, -i$. Also for this transformation find the images of $|z| \leq 1$.

(b) Show that the circle of convergence of the power series $f(z) = \sum_{n=0}^{\infty} z^{n!}$ is a natural boundary.

SECTION—C

5. (a) State and prove Toylor's theorem.

(b) State and prove Maximum modulus principle.

6. (a) If f(z) is analytic within and on a closed contour C except at a finite number of poles and is not zero on C then show that $N - P = \frac{1}{2\pi i} \int_C \frac{f'(z)}{f(z)} dz$, where N is the number of zero and P is the number of poles inside C.

(b) Apply Rouche's theorem to determine the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$ that lie inside the circle $|z| = 1$.

SECTION—D

7. (a) Define residue at infinity. Find the residue of $\frac{z^3}{z^2-1}$ at $z = \infty$.

(b) Evaluate $\int_0^{\pi} \frac{a\, d\phi}{a^2 + \sin^2 \phi}$.

8. (a) Show by contour integration that $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$.

(b) Prove that $\int_0^{\infty} \frac{\log x}{(1+x^2)^2} dx = \frac{\pi}{4}$, using as a contour a large semicircle in the upper half plane indented at the origin.